ONE FORM OF CURRENT INSTABILITY FOR A PLASMA OF FINITE CONDUCTIVITY

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This paper considers whether a specific type of instability associated with pressure and conductivity gradients can exist in a low-temperature dense plasma. Apart from hydromagnetic instabilities [1], specifically low-temperature instabilities associated with energy dissipation can exist in a low-temperature dense plasma (T = 10–100 eV, n = 10^{16}–10^{18} cm^{-3}). Instabilities in a low-temperature, current-carrying plasma as the result of heating and radiation were considered in [2], and stability criteria were obtained on the assumption that temperature gradients, and consequently the conductivity, are small.

The present paper considers whether still another type of instability, associated with conductivity and pressure gradients, can exist in a plasma of this sort. The mechanism by which this instability operates is similar to that of the current-convective [3] and resistive [4] instabilities.

We shall consider a cylindrical column of plasma contained by the magnetic field of the self current, which loses its Joule heat by radiation and thermal conductivity. In what follows standard symbols adopted in [5] are used.

The operating mechanism for the instability can be understood from the following considerations. In the equilibrium state the equations of pressure and energy balance have the form

$$\nabla P = c^{-1}\mathbf{j}\times\mathbf{H}, \qquad \operatorname{rot}\mathbf{H} = 4\pi c^{-1}\mathbf{j}$$
$$\operatorname{div}(\varkappa \nabla T) = -\sigma E^2 + Q_r \tag{1}$$

Here Q_r is the power radiated,

$$\mathbf{j} = \sigma\mathbf{E}, \qquad \sigma = \sigma_1 T^{3/2}/\lambda, \qquad \sigma_1 = \text{const}$$

We consider rapid oscillations when the period of the oscillations is less than the characteristic time for energy dissipation, but larger than the characteristic time for the skin effect τ_s. When the plasma is displaced by δ_r from the equilibrium state as the result of adiabatic compression, the following temperature change occurs:*

$$\delta T = \frac{d\ln T_0}{d\ln P_0}\,\frac{d\ln P_0}{d\ln r}\,\frac{\delta r}{r_0}\,T_0$$

There is consequently a change in conductivity:

$$\delta\sigma = \frac{d\ln\sigma_0}{d\ln T_0}\,\frac{\delta T}{T_0}\,\sigma_0$$

Since the electric field is constant, this leads to a change in the force containing the plasma $\mathbf{j}\times\mathbf{H}$ when it is displaced to the point $r_0 + \delta r$

$$\delta\mathbf{F} = c^{-1}\delta\mathbf{j}\times\mathbf{H}_0 + c^{-1}\mathbf{j}_0\times\delta\mathbf{H}, \quad \delta\mathbf{j} = \delta\sigma\mathbf{E}_0, \quad \operatorname{rot}(\delta\mathbf{H}) = 4\pi c^{-1}\delta\mathbf{j}$$

*Here and in what follows a subscript c denotes quantities connected with surface processes (skin-effects); while a subscript a indicates quantities associated with adiabatic processes.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 11, No. 2, pp. 47–49, March–April, 1970. Original article submitted July 17, 1969.

(the subscript 0 denotes equilibrium quantities).

Since a change in the containing force at the point $r_0 + \delta r$ is determined by a change in conductivity at the point $r_0 + \delta r$ compared with that at the point r_0

$$\delta\sigma = \frac{d\ln\sigma_0}{d\ln r}\frac{\delta r}{r_0}\sigma_0$$

we can neglect the term

$$|j_0 \times \delta H| \sim (k\rho)^{-1}\delta j H_0, \qquad \rho^{-1} = d\ln H_0 / dr$$

for short-wavelength perturbations with $k\rho \gg 1$ and obtain the total change in the containing force (the subscript 0 is omitted in what follows):

$$\delta\mathbf{F} = -\mathbf{j}\times\mathbf{H}\left[-\frac{d\ln\sigma}{d\ln r} + \left(\frac{d\ln\sigma}{d\ln P}\frac{d\ln P}{d\ln r}\right)_a\right]\frac{\delta r}{r} \tag{2}$$

Thus, the equilibrium condition has the form

$$\varphi = -\frac{d\ln\sigma}{d\ln r} + \left(\frac{d\ln\sigma}{d\ln P}\right)_a\frac{d\ln P}{d\ln r} > 0$$

or

$$\frac{d\ln\sigma}{d\ln P} > \left(\frac{d\ln\sigma}{d\ln P}\right)_a \tag{3}$$

By way of an example, now consider the equilibrium condition for the case in which the fundamental cause of energy losses is radiation. The quantity $d\ln\sigma/d\ln P$ can be found by considering the equation of energy balance

$$\sigma E^2 = \alpha n^2 f(T)$$

For low temperatures, when recombination radiation is predominant, we have

$$f(T) = T^{-1/2} \qquad T \sim P^{1/2}, \quad d\ln\sigma/d\ln P = 3/4$$

When bremmstrahlung predominates, we have

$$f(T) = T^{1/2}, \quad T \sim P^{2/3}, \qquad d\ln\sigma/d\ln P = 1$$

Since

$$T_a \sim P^{(\gamma-1)/\gamma}, \qquad d\ln\sigma/d\ln P = 3/5$$

the equilibrium condition (3) is satisfied.

The equilibrium increment can be obtained by direct consideration of the equation of motion for the plasma for a small displacement from the equilibrium position:

$$mn\frac{d^2(\delta r)}{dt^2} = \delta F \tag{4}$$

Inserting δF from Eq. (2) in (4), we obtain

$$\omega^2 = \omega_0^2\alpha_1^2, \quad \alpha_1^2 = \gamma^{-1}|d\ln P/d\ln r|\varphi, \quad \omega_0^2 = \frac{P_0}{mn_0R^2}$$

The equilibrium equation (1), rewritten in the form

$$\frac{d\ln P}{d\ln r} = -\frac{2}{\beta}\left(1 + \frac{d\ln H}{d\ln r}\right)$$

gives us the increment

$$\alpha_1^2 = \frac{1}{\gamma}\left[\frac{2}{\beta}\left(1 + \frac{d\ln H}{d\ln r}\right)\right]^2\left[-\frac{d\ln\sigma}{d\ln P} + \left(\frac{d\ln\sigma}{d\ln P}\right)_a\right]$$

Since the quantities $d\ln H/d\ln r$, $-d\ln\sigma/d\ln P + (d\ln\sigma/d\ln P)_a$ are of the order of unity, then in order of magnitude

$$\alpha_1^2 \sim 4\gamma^{-1}\beta^{-2}$$

and the growth time of the perturbation is

$$\tau \sim 2\beta^{-1}\gamma^{-1/2}\omega_0^{-1}$$

In order for the conditions adopted to be satisfied, the increment must be less than the inverse skin-effect time $\tau^{-1} > \tau_c^{-1}$; otherwise, current diffusion must be considered. This requirement leads to the condition with

$$\sqrt{\beta} < R^3 T_{96}^{-2}$$

For modes with $m \neq 0$ the pattern of instability development is qualitatively not much different. We note that this instability is associated with temperature perturbations, and so the thermal conductivity is the stabilizing factor.

Finally, the author is grateful to S. G. Alikhanova for encouragement and interest in the work and to S. L. Mushera for useful discussions.

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